Kripke Semantics for Fuzzy Logics

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Abstract

Kripke frames (and models) provide a suitable semantics for sub-classical logics; for example Intuitionistic Logic (of Brouwer and Heyting) axiomatizes the reflexive and transitive Kripke frames (with persistent satisfaction relations), and the Basic Logic (of Visser) axiomatizes transitive Kripke frames (with persistent satisfaction relations). Here, we investigate whether Kripke frames/models could provide a semantics for fuzzy logics. For each axiom of the Basic Fuzzy Logic, necessary and sufficient conditions are sought for Kripke frames/models which satisfy them. It turns out that the only fuzzy logics (logics containing the Basic Fuzzy Logic) which are sound and complete with respect to a class of Kripke frames/models are the extensions of the Gödel Logic (or the super-intuitionistic logic of Dummett); indeed this logic is sound and strongly complete with respect to reflexive, transitive and connected (linear) Kripke frames (with persistent satisfaction relations). This provides a semantic characterization for the Gödel Logic among (propositional) fuzzy logics. This logic can be aximatized as the Intuitionistic Logic plus the axiom $(\varphi \to \psi) \vee (\psi \to \varphi)$, and is sound and strongly complete with respect to reflexive, transitive, and connected Kripke frames (with persistent satisfaction relations). Gödel Fuzzy Logic is axiomatized as BL plus the axiom $\varphi \to (\varphi \& \varphi)$ of idempotence of conjunction. Dummett showed that this logic can be completely axiomatized by the axioms of intuitionistic logic plus the axiom $(\varphi \to \psi) \vee (\psi \to \varphi)$. We will show that the only class of Kripke models which could be sound and (strongly) complete for a logic containing BL must contain the

class of reflexive, transitive, connected and persistent Kripke models. In the other words, any logic that contains BL and is axiomatizing a class of Kripke frames/models must also contain the Gödel–Dummett Logic. So, a Kripke-Model-Theoretic characterization of Gödel Fuzzy Logic is that *it is the smallest fuzzy logic containing the Basic Fuzzy Logic which is sound and complete with respect to a class of Kripke frames/models.* Also, the class of reflexive, transitive, connected and persistent Kripke models is the smallest class that can be axiomatized by a propositional fuzzy logic.